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OPTIMAL DESIGN OF PIPELINES AND SPHERICAL TANK

Imre TIMÁR [1], Pál HORVÁTH [2]
[1]Professor, University of Pannonia, timari@almos.vein.hu; www.uni-pannon.hu
[2]Assoc. Prof. University of Pannonia, horvathp@almos.vein.hu; www.uni-pannon.hu

***Abstract*—**The evaluation of optimal insulation has been presented for insulated pipelines and spherical tank. In the majority of the insulation problems the objective function and the restrictions are nonlinear. An efficient nonlinear optimization method was applied for the minimum cost design of insulated pipelines and spherical tank with constrains. The objective (cost) function includes the heat loss and the material costs of the insulating layers and material costs. The thickness of insulating layer and the minimum costs depending on the temperature of surrounding air are presented.

***Keywords*—**heat transfer, insulation optimization, restrictions

I. INTRODUCTION

BETWEEN two bodies that interface, but their temperature is different heat transfer come off. The aim of the heat insulation is to keep the evolving convection at the lowest bench. The reduction of heat loss is the efficient method the heat insulation [1]. The heat insulation is used in the industry at high and low temperature, too. The materials and the methods we are using by heat insulation have to be selected watchfully to the aim. The coefficient of heat conductivity the different materials are very different. In practice we can say that those materials are isolators where the coefficient of heat conductivity is lower than 0.1 (W/mK). These days the most frequently used heat insulation materials are the synthetic organic materials, e. g. polyurethane foam. The polyurethane molecule properties we can be modified with chemical and physical methods so we could reach different properties (e.g. coefficient of heat conductivity, density). Using of polyurethane foam is supported as the readied shape fills and adhesives. Tanks to these properties are made with different thickness. Thus we can do the optimal thickness easily and it could be used within wide temperature threshold limit (from -196^{o}C to +130^{o}C) [2]. That is important, because the heating insulation could be divided to cold and warm. The aim of economical heating isolation is to define the minima of the objective function which consist of the heat waste and the costs of heat insulation.

II. THE OPTIMAL THICKNESS OF HEAT INSULATION OF PIPE LINE

A. The heat transfer and the thermal resistance of heat conduction

In this situation heat is transferred radially through a pipe wall. For the steady-state case the heat flow rate is found from the Fourier's law.

$$\dot{Q} = -\lambda A \frac{dt}{dr} = -\lambda 2\pi r l \frac{dt}{dr}, \tag{1}$$

where λ is the thermal conductivity of the material, A is the area across which heat flows, l is the length of pipe, t is the temperature and r is the radius of the pipe. Integration from $r=r_1$ to $r=r_2$ and $t=t_1$ to $t=t_2$ then gives (Fig. 1).

$$\dot{Q} = \frac{t_1 - t_2}{\frac{1}{2\pi\lambda l}\ln\frac{r_2}{r_1}} = \frac{t_1 - t_2}{\frac{1}{2\pi\lambda l}\ln\frac{d_2}{d_1}}. \tag{2}$$

The name of denominator is the thermal resistance. The thermal resistance of pipe material is

$$R_t = \frac{1}{2\pi\lambda_t l}\ln\frac{d_2}{d_1}, \tag{3}$$

where λ_t is the thermal conductivity of pipe materials.

The thermal resistance of the insulation layer is

$$R_1 = \frac{1}{2\pi\lambda_1 l}\ln\frac{d_2 + 2h_1}{d_2}, \tag{4}$$

where λ_1 is the thermal conductivity of the insulation layer and h_1 the thickness of the insulation layer.

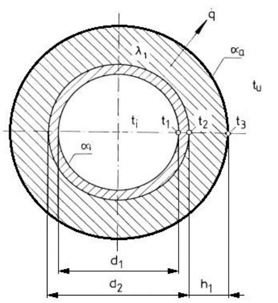

Fig.1. Pipe with insulation

Hot medium temperature in the pipe is t_i and the wall temperature is t_1. The heat transfer occurs between the two elements where the thermal resistance becomes from the Newton's law [3].

$$R_{ai} = \frac{1}{\alpha_i \pi d_1 l}, \quad (5)$$

where α_i is the heat transfer coefficient.

We can calculate the thermal resistance of heat transfer in a similar way between the outside wall surface (t_3) and surroundings temperature (t_u)

$$R_a = \frac{1}{\alpha_a \pi (d_2 + 2h_1) l}, \quad (6)$$

where α_a is the heat transfer coefficient between the cylindrical outside wall surface and the surroundings. Using the heat transfer and the thermal resistance of heat conduction we got the developed heat flux.

$$\dot{Q} = \frac{t_1 - t_u}{R_{ai} + R_t + R_1 + R_a}. \quad (7)$$

After the defining thermal resistance we come to the point of the objective function and the restrictions.

B. Composition of the Objective Function

Our aim is to define the optimal thickness (h_1) of the polyurethane foam insulation while the costs of materials and the heat waste are minimal. In our case the objective function consist of three parts. The cost of pipe material (K_t), material costs of insulation (K_{is}) and the costs of heat waste (K_h).

$$K = K_t + K_{is} + K_h. \quad (8)$$

Members of costs function concerning we take care one meter long, so we calculate with the specific costs.

$$k_t = K_t / l, \quad (9)$$

where l is the length of pipe.

The costs of insulation consist of the cost of insulation layers and the material cost of the cover layer of foam.

$$k_{is} = \frac{\left[(d_2 + 2h_1)^2 - d_2^2\right]\pi}{4} k_1 + \left[(d_2 + 2h_1)\pi + c_a\right] k_a, \quad (10)$$

where c_a is the overlap of cover material (m), k_1 is the specific cost of layer of insulation foam (HUF/m^3) and k_a is the specific material cost of cover layer (HUF/m^2).

The specific heat waste can be calculated with:

$$k_h = \frac{\dot{Q}}{l} \tau k_{hf} \quad (11)$$

formula, where τ is working time of pipe line *s* and k_{hf} is the specific cost of heat (HUF/J).

C. The Restrictions

1. The restriction of heat loss

The real heat loss results from the heat convection from the heat carried medium to the outside surroundings. In this case we reduce the thermal resistance of the alumina surface, because it is small. So the restriction is:

$$\dot{q} = \frac{\dot{Q}}{l} \le \dot{q}_{adm}, \quad (12)$$

where $\dot{q}_{adm}$ is the admissible heat loss.

2. The restriction of the outside surface temperature of the wall

Certain case is necessary to restrict the outside surface temperature of the wall. In this case the allowed temperature is 10 ^{o}C could be higher, than the environmental temperature.

$$t_3 \le t_u + 10. \quad (13)$$

The t_3 temperature is defined from that condition there is the same heat quantity go on in every layer.

$$t_3 = t_i - \frac{(t_i - t_u)(R_{ai} + R_t + R_1)}{R_{aib} + R_t + R_1 + R_a}. \quad (14)$$

D. The Solution of Optimization, Results

Because the heat conduction coefficient is the function of temperature we calculate with the value of the medium temperature. The value of the heat transfer coefficient we take care the connection between in the pipe flowed medium and the inside pipe wall as regards insulation outside surface and the outside surroundings [3], [4].

Data: d_1=0,159 m; d_2=0,168 m; c_a=0,01 m; k_1=5100 HUF/m^3; k_m=600 HUF/m^2; k_t=500 HUF/m; k_{hf}=1.458x10^{-6} Ft/J; l=10 m; $\dot{q}_{adm}$ = 10,…,25 W/m; t_u= -15^{o}C; 0 ^{o}C; 15 ^{o}C; t_i=110 ^{o}C; α_a=48,8 W/m^{2}K; α_i=105,5 W/m^{2}K; λ_1=0,021 W/mK; λ_t=52,3 W/mK; τ=1680 h.

Using the adequate values and connections we made execute program with different parameters.

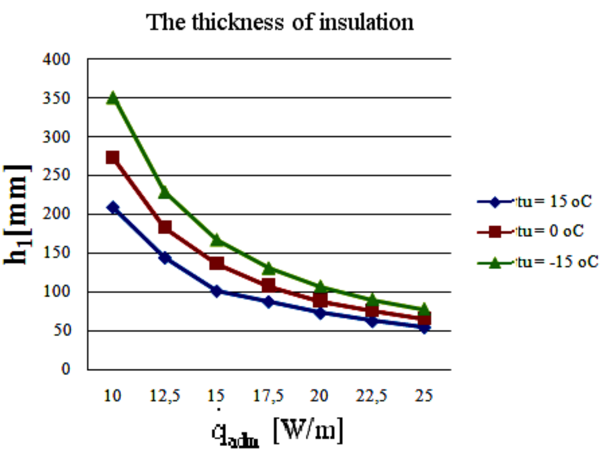

Fig. 2. The optimal thickness of insulation by different temperatures

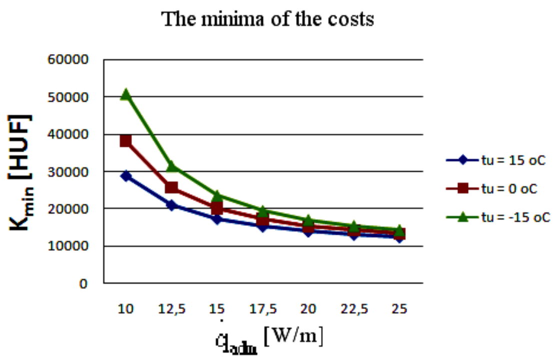

Fig. 3. The minima of the costs by different outside temperature

III. Optimization of Isolation of Spherical Tank

A. Phrasing of the Optimization

Our aim is to define the optimal thickness of insulation foam (h_1) of the spherical vat while the costs of material of insulation layer and heat waste are the lowest. The structural design of the tank is visible in the Fig. 4.

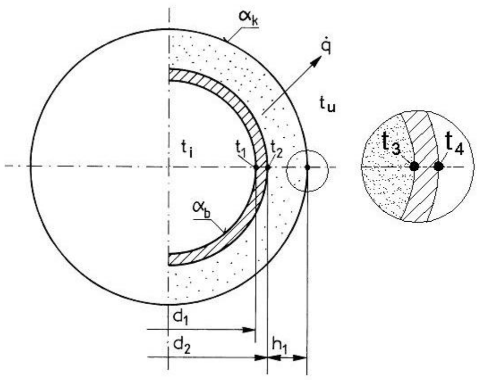

Fig. 4. Structural design of the spherical tank

The surface of the outside insulation covers one alumina sheet that has thickness (v). So the restriction conditions are the maximal heat loss and the maximal temperature of the surface (t_4). The insulation layer is polyurethane foam. By the Fourier's law the heat flux crosses on the surface of the tank in the stationary condition [5]. The spherical tank has *n*-layers insulation.

$$\dot{Q} = \frac{t_1 - t_{n+1}}{\sum_{i=1}^{n} \frac{1}{2\pi\lambda_i}\left(\frac{1}{d_n} - \frac{1}{d_{n+1}}\right)}, \tag{15}$$

where t_1 and t_{n+1} are the temperature of the inside and outside wall of the spherical tank, λ_i is heat conductor coefficient each layer and d_n is an inside sphere diameter which belongs to the *n* layer. The thermal resistance is the wall of the spherical tank

$$R_w = \frac{1}{2\pi\lambda_f}\left(\frac{1}{d_1} - \frac{1}{d_2}\right), \tag{16}$$

where λ_f is heat conduct coefficient of the wall. The thermal resistance of the heat insulation layer

$$R_{is} = \frac{1}{2\pi\lambda_{is}}\left(\frac{1}{d_2} - \frac{1}{d_2 + 2h_1}\right), \tag{17}$$

where λ_{is} is heat conduct coefficient of the heat insulation layer. The thermal resistance of the alumina sheet which covered the insulation layer is

$$R_{al} = \frac{1}{2\pi\lambda_{al}}\left(\frac{1}{d_2 + 2h_1} - \frac{1}{d_2 + 2h_1 + 2v}\right), \tag{18}$$

where λ_{al} is heat conduct coefficient of the alumina. The heat transfer occurs between the medium in the tank and inside surface of the wall is both the outside surface of the insulation layer and the surroundings is happened. The resistance of thermal heat transfer could be calculated [5]. So the resistance of thermal heat transfer is on the inside wall of the tank

$$R_i = \frac{1}{\alpha_i \pi d_1^2}, \tag{19}$$

where α_i is the heat transfer coefficient on the inside wall. The thermal resistance of heat transfer between the surface of heat insulation and surroundings:

$$R_{au} = \frac{1}{\alpha_{au}\pi(d_2 + 2h_1 + 2v)^2}, \tag{20}$$

where α_{au} is coefficient of heat transfer on the surface of the insulation layer and *v* is the thickness of alumina. So with the thermal resistances we can calculate the heat flux:

$$\dot{Q} = \frac{t_i - t_u}{R_i + R_w + R_{is} + R_{al} + R_{au}}, \tag{21}$$

where t_i is the temperature of media in the tank and t_u is the temperature of the air in the surrounding.

B. Composition of the Objective Function

The objective function consists of the cost of material of the insulation layer, the cost of material of alumina cover and the cost of heat waste. The cost of alumina sheet can be easily calculated if we know the diameter of the tank. Knowing the volume of foam and the cost specific material the cost of insulation can be calculated. The heat waste is calculated with knowing of the heat loss ($\dot{Q}$), the specific heat cost (k_{hf}) and working time. On the basis of the above mentioned we can compose the objective function:

$$K = \frac{(d_2 + 2h_1)^3\pi - d_2^3\pi}{6} k_{sz} + \dot{Q} k_{hf} \tau + (d_2 + 2h_1)^2 \pi k_b, \tag{22}$$

where k_{sz} is the specific material cost of polyurethane and k_b is the specific material cost of alumina cover.

C. The Restrictions

1. The restriction of heat loss

We limited the heat loss in the tanks similarly as in the pipe. In this case we prescribe that the heat loss will be smaller than allowed.

$$\dot{Q} \le \dot{Q}_{adm}, \qquad (23)$$

where $\dot{Q}_{adm}$ is the admissible heat loss.

2. The restriction of the temperature of the surface of isolated tank

We determined the temperature of the insulated surface of the tank (t_4) from the condition that the same amount of heat goes through each layers.

$$\dot{Q} = \frac{t_i - t_3}{R_i + R_w + R_{is} + R_{al}} = \frac{t_i - t_u}{R_i + R_w + R_{is} + R_{al} + R_{au}}, \qquad (24)$$

from this the wanted temperature is:

$$t_4 = t_i - \frac{(t_i - t_u)(R_i + R_w + R_{is} + R_{al})}{R_i + R_w + R_{is} + R_{al} + R_{au}}. \qquad (25)$$

In our case we prescribe that the temperature of the surface of tank will be maximum 10 degrease of Celsius warmer than that of the entourage:

$$t_4 \le t_u + 10. \qquad (26)$$

D. The Solution of Optimization, Results

As the next step we define the optimal thickness of the insulation layer of a spherical tank standing in free space. The objective function and the restrictions are as above. As the heat transfer coefficient depends on the temperature and in the calculation we take its value in to consideration. The heat transfer coefficients of the inside and outside of the tank were calculated by the [6] reference. The value of heat transfer coefficient were counted with the average temperature by the calculations.

Data: d_1=3,5 m; d_2=3,516 m; k_{hf}=4.374x10^{-6} HUF/J; k_{sz}=5100 HUF/m^3; k_b=500 HUF/m^2t_b=130°C; τ=2160 h,

t_k=15°C; 0 °C; -15 °C; $\dot{Q}_{adm}$ =200, ,600 W; α_i=2400 W/m^{2}K; α_a=34 W/ m^{2}K; λ_f=52,3 W/mK; λ_{sz}=0,021 W/mK.

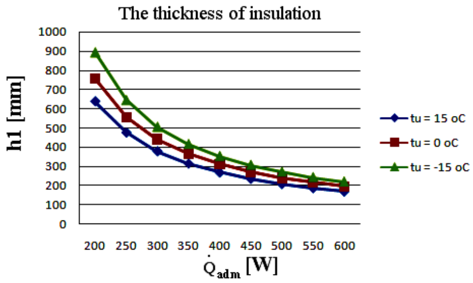

Fig. 5. The optimal thickness of isolations by different temperatures.

In the next we examine how change the value objective function and the thickness of insulation layer when the temperature of surrounding air is changing (t_u=-15°C; 0 °C; 15 °C).

The Figure 5. shows the changing of thickness of insulating layer in function of heat loss (t_i=130 °C). May be seen that the surrounding air temperature is decreased and the thickness of insulation layer grows.

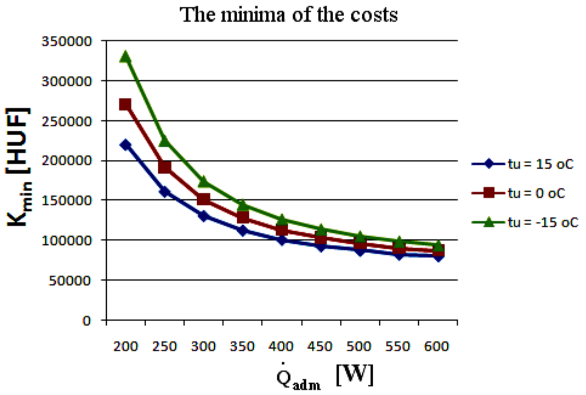

Fig. 6. The minima of the costs by different outside temperature

The Fig. 6 shows the changing of the minima of objective function dependence of heat loss than the temperature of medium is 130 degrease of Celsius in the tank. If we leave higher heat loss the cost is decreased. The reason why in this case the material cost makes determine role in the objective function. From the Fig. 6 is prominent that the temperature of surrounding air is decreased (15 °C; 0 °C; -15 °C) the cost grows. The cause of the growing of heat loss and heat insulations costs are.

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